

Unit 4: Logarithms ~ Learning Guide

Name: _____

Instructions:

Using a pencil, complete the following questions as you work through the related lessons.

Show ALL work as it is explained in the lessons. Do your best and ask your instructor if you don't understand any questions!

Lesson One

The Exponent Laws

1. Simplify the following and write answers with positive exponents only.

a) $\frac{(x^2)^4}{x^{-3}}$

b) $\frac{(m^2n^3)^{-1}}{m^3n}$

c) $\frac{(2x)^{-2}}{(3x^2)^3}$

d) $\left(\frac{x^2}{y}\right)^2 \left(\frac{2y^3}{x}\right)^{-1}$

e) $\left(\frac{9y^6}{4x^2}\right)^{1/2} \div xy$

f) $\frac{(3m)^{-1}}{(2n)^2} \left(\frac{n}{m^2}\right)^3$

g) $(xy)^2 \left(\frac{1}{2x^3}\right)^{-2} \div y$

h) $\left(\frac{(2m^2)^{-1}}{n^3}\right)^2 \div \left(\frac{m}{3n^3}\right)^4$

2. Evaluate the expressions for $x = -1$, $y = 2$, $z = 3$.

a) $(3x^2y^{-1}z^2)(4x^{-1}y^2z)$

b) $\frac{(2xy^3z^{-1})^2}{x^2y^4}$

Lesson Two

Solving Equations Involving Exponents

1. Solve for x .

a) $x^{2/3} = 4$

b) $x^{-1/2} = 9$

c) $5x^{3/2} = 40$

d) $(4x+1)^{2/3} = 25$

2. Solve for x by converting to common bases, show all your work.

a) $2^x = 32$

b) $\left(\frac{1}{3}\right)^x = 81$

c) $5^x = 625$

d) $3^{x-2} = 27$

e) $4^{2x} = 8$

f) $27^{x-1} = 9$

g) $3 \cdot 16^{x+2} = 96$

h) $25^{2x-1} = 5^x$

i) $2 \cdot 8^{x+1} = 16$

j) $4^x = 8^{x+1}$

k) $3^{3x+4} = 9^x$

l) $\left(\frac{1}{4}\right)^{-x+2} = 8^{3x-1}$

Lesson Three

Defining a Logarithm and Logarithmic Restrictions

1. Convert the following exponential equations to logarithmic form.

a) $4^0 = 1$

b) $5^2 = 25$

c) $9^{1/2} = 3$

d) $a^b = c$

2. Convert the following logarithmic equations to exponential form.

a) $\log_3 9 = 2$

b) $\log_4 \frac{1}{16} = -2$

c) $\log 0.1 = -1$

d) $\log_a b = c$

3. Evaluate each of the following.

a) $\log 1000$

b) $\log_2 16$

c) $\log_9 \frac{1}{3}$

d) $\log_2 8\sqrt{2}$

4. Solve for x .

a) $\log_4 64 = x$

b) $\log_x 9 = 2$

c) $\log_5 x = 3$

d) $\log_x 27 = \frac{3}{2}$

5. State the domain for each of the following.

a) $y = \log_3 (x + 5)$

b) $y = \log_2 (2x - 3)$

c) $y = \log_x (3 - x)$

Lesson Four

Laws of Logarithms

1. Write the following as a single logarithm.

a) $\log x + \log y$

b) $2\log_3 x - \log_3 y$

c) $\log x + 5\log y - 3\log x$

d) $-\log x + \frac{1}{2}\log y + 1$

e) $\frac{1}{4}\log_2 x + 3\log_2 y - 3$

2. Expand.

a) $\log \frac{x}{y^2}$

b) $\log_2(\sqrt{x}y)$

c) $\log\left(\frac{y}{x}\right)^2$

d) $\log \frac{100x}{z^2\sqrt[5]{y}}$

e) $\log_2 \frac{\sqrt[3]{x^2y}}{16}$

3. Given that $\log_a x = 2$ and that $\log_a y = 3$ then evaluate the following.

a) $\log_a x^2 y$

b) $\log_a \left(\frac{ay}{x} \right)$

c) $3\log_a (\sqrt{xy})$

4. Simplify each of the following.

a) $5^{2\log_5 x}$

b) $\frac{1}{\log_3 2} + \frac{1}{\log_5 2}$

c) $8^{\log_2 x}$

5. Given that $\log 2 = a$ and that $\log 3 = b$ then determine an expression for the following in terms of a and b without using a calculator.

a) $\log 6$

b) $\log \left(\frac{30}{16} \right)$

c) $\log 1200$

6. If $\log_3 x = 10$ then evaluate each of the following without a calculator.

a) $\log_3 (9x)$

b) $\log_3 \left(\frac{27}{x} \right)$

c) $\log_3 \left(\frac{x^2}{3} \right)$

7. Given that $\log_9 4 = a$ and $\log_{81} 5 = b$, find an expression for $\log_3 20$ in terms of a and b without using a calculator.

Lesson Five

Solving Exponential and Logarithmic Equations & Introduction to Exponential Functions

1. Solve the following exponential equations for the exact value of x then evaluate your solution rounding to 4 decimal places.

a) $3^x = 11$

b) $5^{x-1} = 7$

c) $7^{3x} + 3 = 21$

d) $1 = 2 \cdot 4^x - 5$

e) $3^{x+1} = 5^x$

f) $5 \cdot 2^{3x-2} = 9^x$

g) $6^{2x+1} = 7^{1-x}$

2. Solve the following algebraically for x .

a) $\log_2 x = 0$

b) $\log(x-2) = 1$

c) $\log 15 - \log 5 = \log x$

d) $\log_3(x-2) - \log_3 2 = 3$

e) $\log x + \log x = \log 36$

f) $2\log_3(1-x) = 6$

g) $\log(x-1) + \log(x+2) = 1$

h) $\log(2x+1) = 1 + \log(1-x)$

i) $\log_2(x^2 - 8) = 3$

j) $\log_3(x + 5) = 1 - \log_3(x + 3)$

k) $\log_2(x - 3) - \log_2 5 + \log_2(x - 2) = 2$

l) $\log_2(\log_5 25) = x$

Lesson Six, Seven and Eight

Applications of Exponential Functions

1. Sarah invests \$1200 at a rate of 5% per year compounded monthly.
 - a) How much money would Sarah have after 7 years?

 - b) How long does it take for the investment to grow to \$2000?

2. An investment of \$3000 lost 4% per year compounded weekly.
 - a) How much is the investment worth after 5 years?

 - b) How long does it take for the investment to be reduced to half of the initial amount?

7. The half-life of a certain radioactive isotope is 20 days. How long does it take for 50g of the isotope to decay to 10g?
8. A radioactive substance is produced from nuclear fallout. If 600g of the substance decays to 250g in 20 years what is its half-life?

9. How many times more powerful is an earthquake with a Richter Scale reading of 7.9 than an earthquake with a rating of 3.2?

10. An earthquake of 6.7 occurred off the coast of British Columbia. A few hours later an aftershock occurred that was 150 times less intense as the original earthquake. What was the magnitude of the aftershock?

11. If celery has a pH of 5.9 then how many times more acidic are apricots if they have a pH of 4.1?

12. If egg yolks have a pH of 6.4 then what is the pH of egg whites if they are 63 times more alkaline?

Lesson Nine and Ten

Graphs of Exponential and Logarithmic Functions

1. Basic exponential functions $y = a^x$ have a horizontal asymptote of the form $y = 0$.
Which transformations (translation, reflection, expansion/compression), when applied to the equation, will change the:

a) equation of the asymptote _____

b) range _____

2. Basic logarithmic functions $y = \log_a x$ have a vertical asymptote of the form $x = 0$.
Which transformations (translation, reflection, expansion/compression), when applied to the equation, will change the:

a) equation of the asymptote _____

b) domain _____

3. Determine the equation of the asymptote in the following functions.

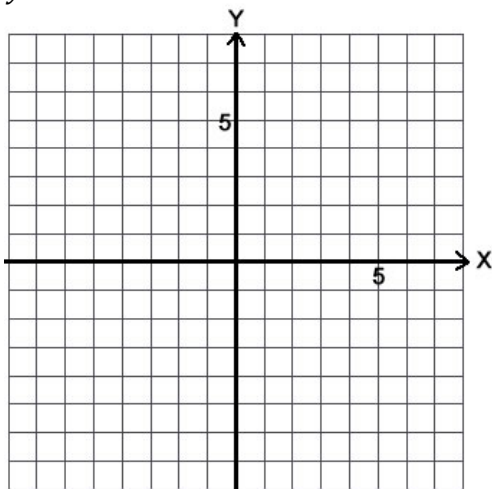
a) $y = 5^{x+1} - 2$

b) $y = \log(x + 4) - 3$

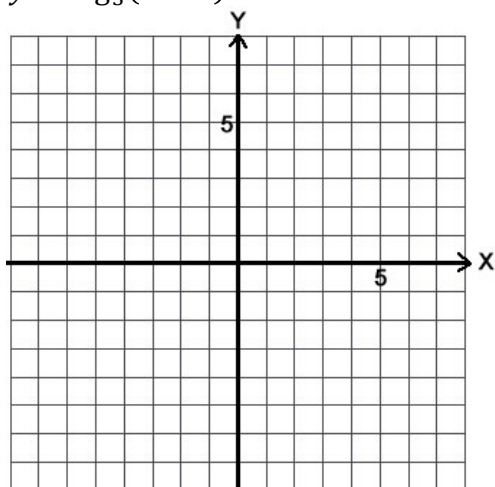
c) $y = -3^{2x} + 4$

4. Graph the following equations by plotting at least 3 points on the curve. Explicitly state the equation of the asymptote, the x -intercept, the y -intercept, and the domain and range.

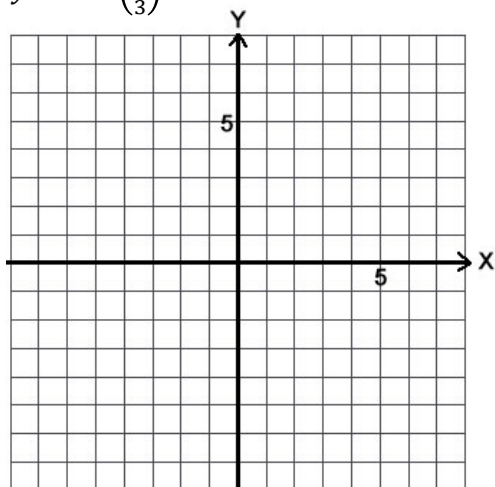
a) $y = 2^{x-3} - 4$



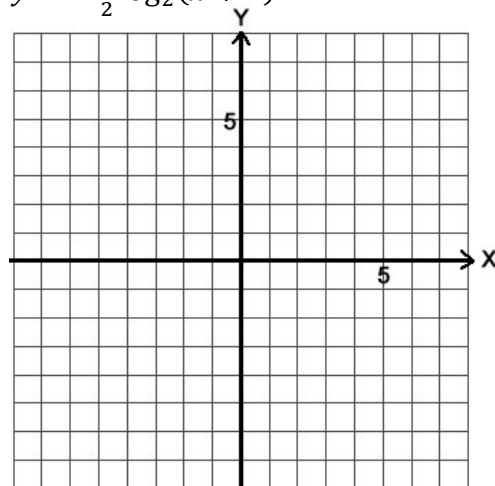
b) $y = \log_3(x + 2) + 1$



c) $y = -3\left(\frac{1}{3}\right)^{-x} + 6$



d) $y = -\frac{1}{2}\log_2(x + 4) - 2$



ANSWERSLesson One The Exponent Laws

1. a) x^{11} b) $\frac{1}{m^5n^4}$ c) $\frac{1}{108x^8}$ d) $\frac{x^5}{2y^5}$ e) $\frac{3y^2}{2x^2}$ f) $\frac{n}{12m^7}$ g) $4x^8y$ h) $\frac{81n^6}{4m^8}$ 2. a) -648 b) $\frac{16}{9}$

Lesson Two Solving Equations Involving Exponents

1. a) 8 b) $\frac{1}{81}$ c) 4 d) 31
 2. a) 5 b) -4 c) 4 d) 5 e) $\frac{3}{4}$ f) $\frac{5}{3}$ g) $-\frac{3}{4}$ h) $\frac{2}{3}$ i) 0 j) -3 k) -4 l) $-\frac{1}{7}$

Lesson Three Definition of a Logarithm

1. a) $\log_4 1 = 0$ b) $\log_5 25 = 2$ c) $\log_9 3 = \frac{1}{2}$ d) $\log_a c = b$
 2. a) $3^2 = 9$ b) $4^{-2} = \frac{1}{16}$ c) $10^{-1} = 0.1$ d) $a^c = b$
 3. a) 3 b) 4 c) $-\frac{1}{2}$ d) $\frac{7}{2}$ 4. a) 3 b) 3 c) 125 d) 9
 5. a) D: $-5 < x$ b) D: $\frac{3}{2} < x$ c) D: $0 < x < 3, x \neq 1$

Lesson Four Laws of Logarithm

1. a) $\log(xy)$ b) $\log_3\left(\frac{x^2}{y}\right)$ c) $\log\left(\frac{y^5}{x^2}\right)$ d) $\log\left(\frac{10\sqrt{y}}{x}\right)$ e) $\log_2\left(\frac{\sqrt[4]{xy^3}}{8}\right)$
 2. a) $\log x - 2 \log y$ b) $\frac{1}{2} \log_2 x + \log_2 y$ c) $2 \log y - 2 \log x$
 d) $2 + \log x - 2 \log z - \frac{1}{5} \log y$ e) $\frac{2}{3} \log_2 x + \frac{1}{3} \log_2 y - 4$
 3. a) 7 b) 2 c) 12 4. a) x^2 b) $\log_2 15$ c) x^3
 5. a) $a + b$ b) $1 + b - 4a$ c) $2a + b + 2$ 6. a) 12 b) -7 c) 19
 7. $2a + 4b$

Lesson Five and Six Solving Exponential and Logarithmic Equations & Introduction to Exponential Functions

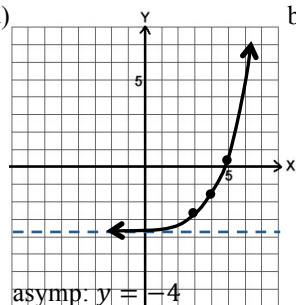
1. Exact answers may vary a) $\frac{\log 11}{\log 3} \cong 2.1827$ b) $\frac{\log 5 + \log 7}{\log 5} \cong 2.2091$ c) $\frac{\log 18}{3 \log 7} \cong 0.4951$
 d) $\frac{\log 3}{\log 4} \cong 0.7213$ e) $\frac{\log 3}{\log 5 - \log 3} \cong 2.1507$ f) $\frac{2 \log 2 - \log 5}{3 \log 2 - \log 9} \cong 1.8945$ g) $\frac{\log 7 - \log 6}{2 \log 2 + 2 \log 3 + \log 7} \cong 0.0279$
 2. a) 1 b) 12 c) 3 d) 56 e) 6 f) -26 g) 3 h) $\frac{3}{4}$ i) ± 4 j) -2 k) 7 l)

Lesson Seven and Eight Applications of Exponential Functions

1. a) \$1701.64 b) 10.24 years 2. a) \$2456.00 b) 17.32 years 3. 3.7% 4. 70.13 years
 5. 39.43m 6. 7.92wks 7. 46.44 days 8. 15.83 years 9. 50118.72 10. 4.52
 11. 63.10 12. 8.2

Lesson Nine and Ten Graphs of Exponential and Logarithmic Functions

1. a) vertical translation b) vertical translation and reflection in the x -axis
2. a) horizontal translation b) horizontal translation and reflection in the y -axis
3. a) $y = -2$ b) $x = -4$ c) $y = 4$
4. a)

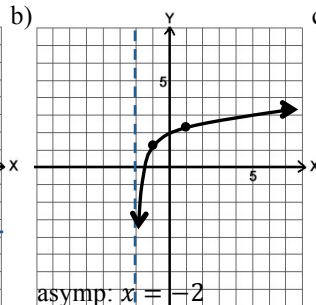


$$x = 5$$

$$y = -\frac{31}{8}$$

$$D: x \in \mathbb{R}$$

$$R: -4 < y$$

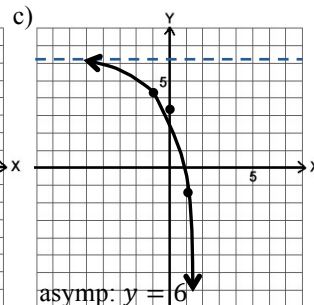


$$x = -\frac{5}{3}$$

$$y = \log_3 2 + 1$$

$$D: -2 < x$$

$$R: y \in \mathbb{R}$$

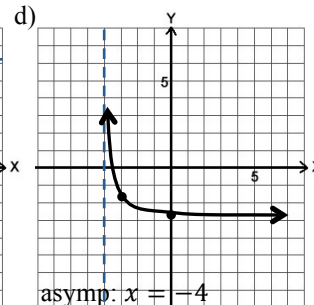


$$x = -\log_{\frac{1}{3}} 2$$

$$y = 3$$

$$D: x \in \mathbb{R}$$

$$R: y < 6$$



$$x = -\frac{63}{16}$$

$$y = -3$$

$$D: -4 < x$$

$$R: y \in \mathbb{R}$$